

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES FPGA IMPLEMENTATION OF ELLIPTIC CURVE DISCRETE LOGARITHM USING VERILOG HDL

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ABSTRACT

Elliptic Curve Discrete Logarithm (ECDL) are most popular choice Elliptic Curve Cryptography (ECC), which gives provision for shorter key lengths as compared to its counterpart public key cryptosystems, and it can be used for security in embedded systems, wireless communications and personal communication systems. In this paper Elliptic Curve Discrete Logarithm code has been written in Verilog Hardware Description Language (HDL) and implemented on Xilinx Spartan3E Field Programmable Gate Array (FPGA), has taken 403 encoders, decoders with minimum period of 5.043 ns, maximum frequency 198.295 MHz and with a total memory usage of 269824 Kilobytes respectively. The performance of the crypto system is much faster than the software implementation of the same system.

Keywords: *Elliptic Curve Discrete Logarithm (ECDL), Verilog Hardware Description Language (HDL), Elliptic Curve Cryptography (ECC), Field Programmable Gate Array (FPGA), Finite Fields.*

I. INTRODUCTION

The idea of information security lead to the evolution of Cryptography. In other words, Cryptography is the science of keeping information secure. It involves encryption and decryption of messages. Encryption is the process of converting a plain text into cipher text and decryption is the process of getting back the original message from the encrypted text. Cryptography, in addition to providing confidentiality, also provides Authentication, Integrity and Non-repudiation. There have been many known cryptographic algorithms.

Based on the key, cryptosystems can be classified into two categories: *Symmetric* and *Asymmetric*. In Symmetric Key Cryptosystems, we use the same key for both Encryption as well as the corresponding decryption. i.e. if K was the key and M was the message, then, we have $D_K(E_K(M)) = M$. Asymmetric or Public key or shared key cryptosystems use two different keys. One is used for encryption while the other key is used for decryption. The two keys can be used interchangeably. One of the keys is made public (shared) while the other key is kept a secret. i.e. let k_1 and k_2 be public and private keys respectively. Let M be the message, then $D_{k_2}(E_{k_1}(M)) = D_{k_1}(E_{k_2}(M)) = M$. In general, symmetric key cryptosystems are preferred over public key systems due to the following factors:

1. Ease of computation
2. Smaller key length providing the same amount of security as compared to a larger key in Public key systems.

The idea of using Elliptic curves in cryptography was introduced by Victor Miller and Neal Koblitz as an alternative to established public-key systems such as DSA and RSA. The Elliptical curve Discrete Log Problem (ECDLP) makes it difficult to break an ECC as compared to RSA and DSA where the problems of factorization or the discrete log problem can be solved in sub-exponential time. This means that significantly smaller parameters can be used in ECC than in other competitive systems such as RSA and DSA. This helps in having smaller key size hence faster computations.

Elliptic Curve Discrete Logarithm Problem:

The strength of the Elliptic Curve Cryptography lies in the Elliptic Curve Discrete Log Problem (ECDLP). The statement of ECDLP is as follows.

Let E be an elliptic curve and $P \in E$ be a point of order n . Given a point $Q \in E$ with $Q = mP$, for a certain $m \in \{2, 3, \dots, m - 2\}$. Find the m for which the above equation holds.

When E and P are properly chosen, the ECDLP is thought to be infeasible. Note that $m = 0, 1$ and $m - 1$, Q takes the values O, P and $-P$. One of the conditions is that the order of P i.e. n be large so that it is infeasible to check all the possibilities of m .

The difference between ECDLP and the Discrete Logarithm Problem (DLP) is that, DLP though a hard problem is known to have a sub exponential time solution, and the solution of the DLP can be computed faster than that to the ECDLP. This property of Elliptic curves makes it favorable for its use in cryptography.

Elliptic Curves: An Elliptic Curve is a set of point on a curve $y^2 = x^3 + ax + b$ given certain real numbers a and b . For example **Figure 1**.

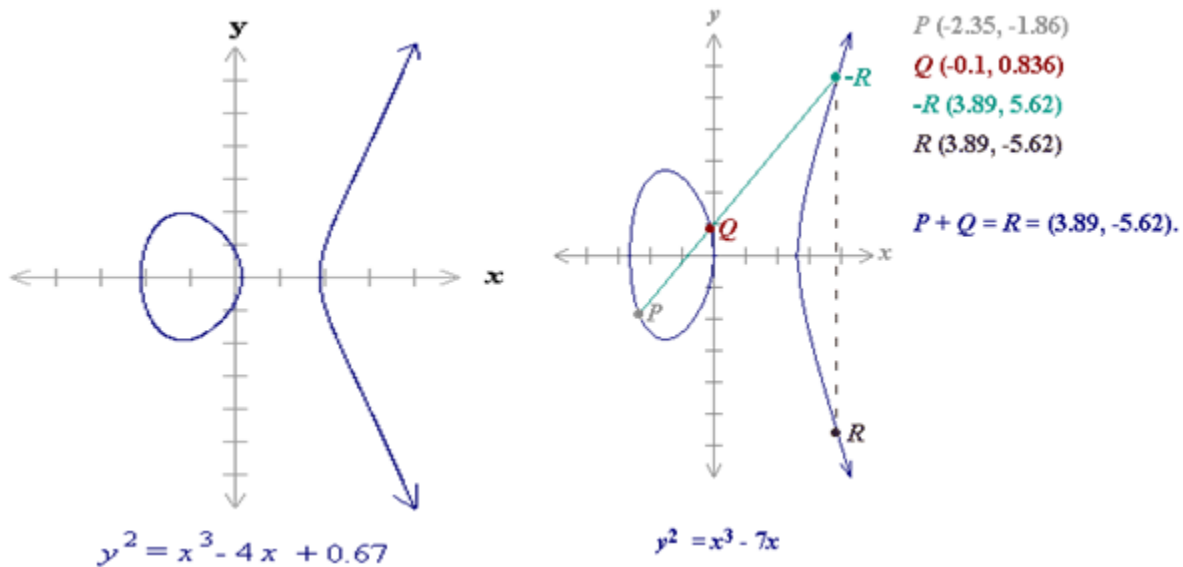


Figure 1: An Elliptic Curve Defined Over \mathbb{R} Figure 2: Adding Two Points on an Elliptic Curve

Elliptic Curve Groups: The set of points on an elliptic curve, plus a special point ∞ form an additive group. The addition of two points on an elliptic curve is defined geometrically, as shown in the **Figure 2**.

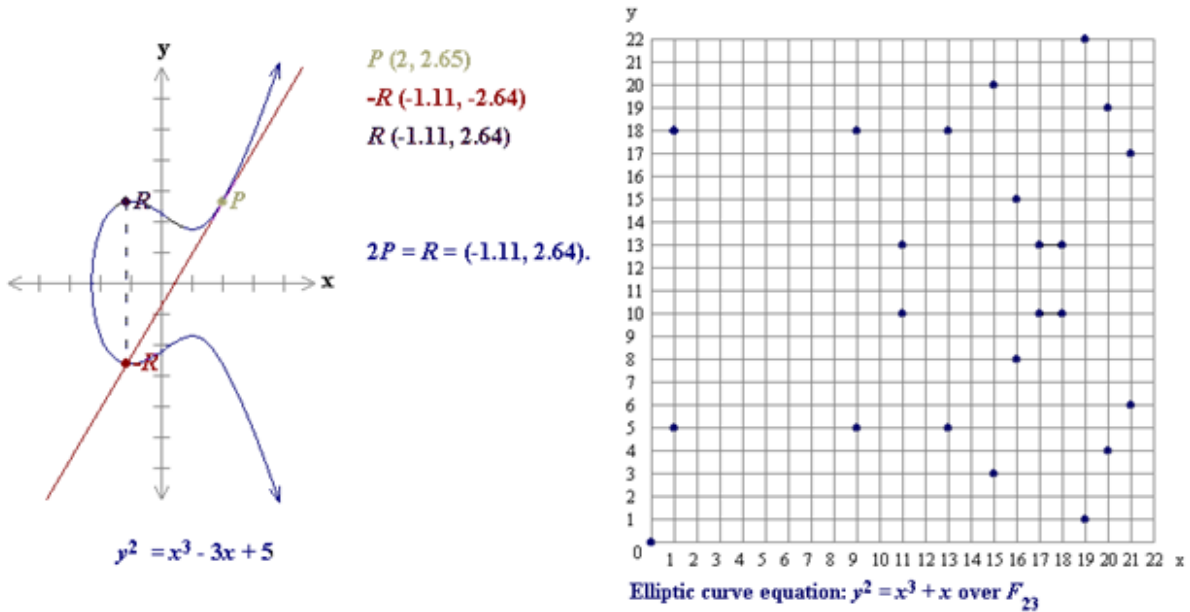


Figure 3 : Doubling a Point on an Elliptic Curve Figure 3 : Elements of this ECG

Elliptic Curve Encryption Algorithms depend on the difficulty of calculating k^P where k a product of two large primes is and P is an element in the Elliptic Curve Group. Geometrically to add a point P to itself you first construct the tangent line to the curve at the point. Then the line will intersect the curve at only one point, and the addition of $2P$ is then defined to be the negative of the point of intersection as seen in figure. Elliptic curve groups over real numbers are not practical for cryptography due to slowness of calculations and round-off error. This Elliptic Curves Over Finite Fields are used. An elliptic curve over a finite field F_p of characteristic greater than three can be formed by choosing the variables a and b within the field F_p .

Roughly speaking the elliptic curve is then the set of points (x, y) which satisfy the elliptic curve equation $y^2 = x^3 + ax + b$ mod p , where $x, y \in F_p$; together with a special point α . If $x^3 + ax + b$ contains no repeated factors, or equivalently if $4a^3 + 27b^2 \neq 0 \pmod{p}$, then these points form a group.

It is well known that ECG (the Elliptic Curve Group) is an additive abelian group with α serving as its identity element.

Example: In the ECG of $y^2 = x^3 + x^2$ over the field F_{23} the point $(9, 5)$ satisfies the equation $y^2 = x^3 + x^2 \pmod{23}$ as $25 \equiv 729 + 9 \pmod{23}$. The elements of this ECG are given in the pictured above.

Obviously we no longer have a curve to define our addition geometrically. Emulating the geometric construction for addition, the formulas for addition over F_p (characteristic 3) are given as follows: Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be elements of the ECG. Then $P + Q = (x_3, y_3)$, where

$$x_3 = \lambda^2 \text{ and } x_1 - x_2 \qquad y_3 = \lambda(x_1 - x_3) - y_1 \qquad \lambda = \begin{cases} \frac{y_2 - y_1}{x_2 - x_1} & \text{if } P \neq Q \\ \frac{3x_1^2 + a}{2y_1} & \text{if } P = Q \end{cases}$$

Definition of the Discrete Logarithm Problem:

In the multiplication group \mathbb{F}_p^* , the discrete logarithm problem that is: Given elements r and q in \mathbb{F}_p^* , find a number k such that $r = qk \pmod{p}$.

Similarly the Elliptic Curve Discrete Logarithm Problem is: Given points P and Q in an ECG over a finite field find an integer k such that $Pk = Q$. Here k is called the discrete log of Q to the base P .

This doesn't seem like a difficult problem, but if you don't know what k is calculating $Pk = Q$ takes roughly $2^{k/2}$ operations. So if k is say, 160 bits long, then it would take about 2^{80} operations!! To put this into perspective, if you could do a billion operations per second, this would take about 38 million years. This is a huge savings over the standard public key encryption system where 1024 and 3074 bit keys are recommended. The smaller size of the keys for Elliptic Curve Encryption makes it ideal for applications such as encrypting cell-phone calls, credit card transactions, and other applications where memory and speed are an issue. There are pros and cons to both ECC and RSA encryption. ECC is faster than RSA for signing and decryption, but slower than RSA for signature verification and encryption. Much of the material used in this paper can be found in the websites listed in the references.

II. OBJECTIVE OF THE PAPER

The objective of this paper is to design and implement an efficient Elliptic Curve Cryptography algorithm for security in data transfer using Verilog HDL. The algorithm is simulated, synthesized and implemented on FPGA. HDL based algorithm occupies less memory space for execution and the execution speed of HDL is very high when compared to the algorithm implemented using high level language. The Verilog HDL language based for hardware implementation of the ECC provides better reliability, than software based security system. The reconfiguration and recustomisation becomes easier at any stage.

III. PROPOSED WORK

The growing possibility of modern communications needs special means of security especially on computer network. The network security is becoming more important as the number of data being exchanged on the internet increases. Therefore, the confidentiality and data integrity are required to protect against unauthorized access. This resulted in an explosive growth in the field of cryptography. All the most popular forms of security protection for information hiding rely on cryptographic techniques which encrypt the private data so that it cannot be read by anyone except the intended recipient who has the right key to convert the encrypted message back to its original form. In this paper information security is done by implementing Elliptic curve cryptography in Verilog HDL. The implementation in Verilog HDL consists of three main modules namely,

1. Main Controller
2. Multiplier and
3. Adder

The program written in Verilog HDL run by Xilinx ISE tool when synthesized gives the hardware module of the program up to the gate level and then given for implementation using FPGA which does the chip layout. Before implementation the program is verified by using simulation which gives the output as signal waveform and even the test bench program can be written and verified using simulation software before synthesis using Xilinx ISE.

IV. MAIN MODULES OF ELLIPTIC CURVE CRYPTOGRAPHY :

The basic unit of description in Verilog is the module. A module describes the structure or functionality of a design and also describes the port through which it communicates externally with the other modules. The structure of a design is described using switch-level primitives, gate-level primitives and user defined primitives. Dataflow behaviour of a design is described using continuous assignments and sequential behaviour is described using procedural constructs.

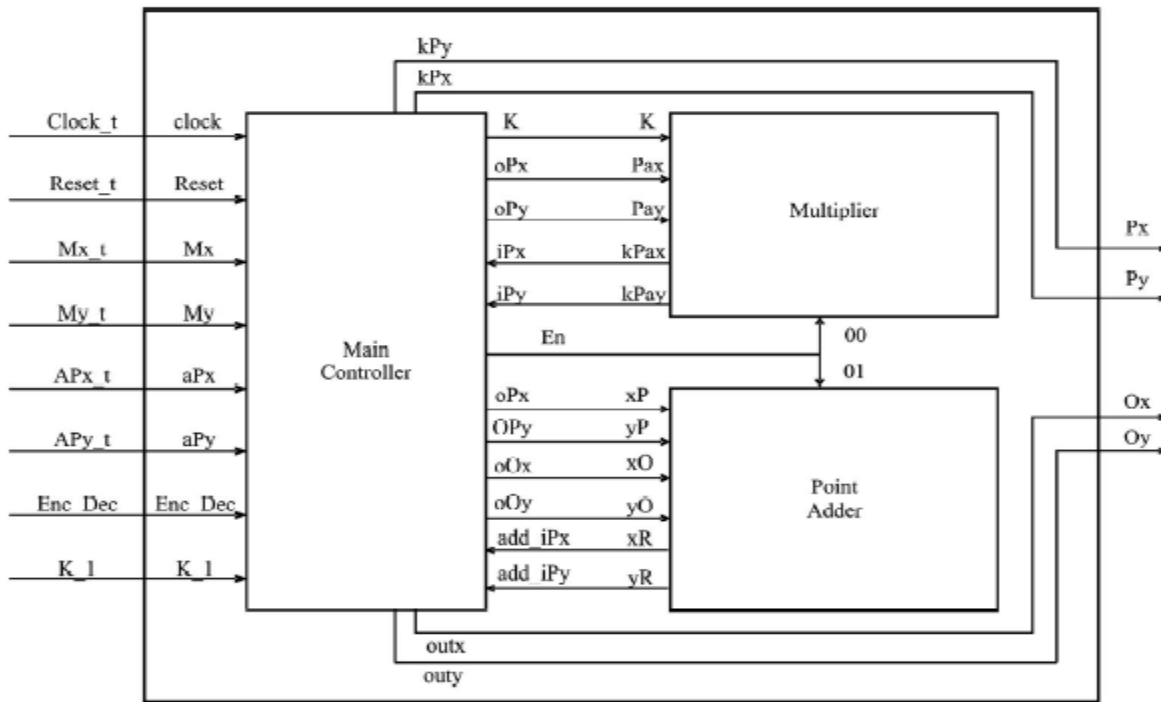


Figure 5 : Main Modules in Elliptical Curve Cryptography

Figure 5 shows the above three modules. The main controller controls the functioning of the adder and multiplier components. The multiplier block is selected when the Enable line is '00'. The multiplier performs multiplication of an integer with a point on the elliptic curve. The multiplication is done by successive addition. The adder block is selected when the Enable line is '01'. The adder performs addition of two points on an elliptic curve.

Figure 6 shows Block Diagram of Main Controller, the main controller controls the functioning of the adder and multiplier components. It has several internal signals, namely **Clock**: The internal clock; **Reset**: The reset signal is used to bring back all the components to their initial conditions, when set to '1'. **Mx** : X -coordinate of the message to be transmitted ; **My** : Y - coordinate of the message to be transmitted; **aPx** : X - coordinate of the quantity "xP", (required in the encrypter part); **aPy** : Y - coordinate of the quantity "yP", (required in the encrypter part); **Enc_Dec** : Selects encryption/decryption; '0' – Encryption; '1' – Decryption; **k_l** : A very large integer (Private key) generated at random; **En** : Enables Multiplier/point Adder; '00' – Multiplier; '01' – Point Adder.

To Multiplier: **k**: A very large integer generated at random which is multiplied with the point; **oPx** : X - coordinate of the point which is to be multiplied with the integer; **oPy** : Y - coordinate of the point which is to be multiplied with the integer.

To Point Adder: **oPx**: X - coordinate of the addend; **oPy**: Y - coordinate of the addend; **oQx** : X - coordinate of the augend; **oQy** : Y - coordinate of the augend.

From Multiplier: **iPx** : X - coordinate of the result; **iPy** : Y - coordinate of the result;

From Point Adder: **add_iPx**: X - coordinate of the result; **add_iPy**: Y - coordinate of the result;

Final Outputs: **kPx**: X - coordinate of the product "xP" (Encryption); **kPy**: Y - coordinate of the product "yP" (Encryption); **outx**: X - coordinate of the expression "xyP+M"; **outy**: Y - coordinate of the expression "xyP+M".

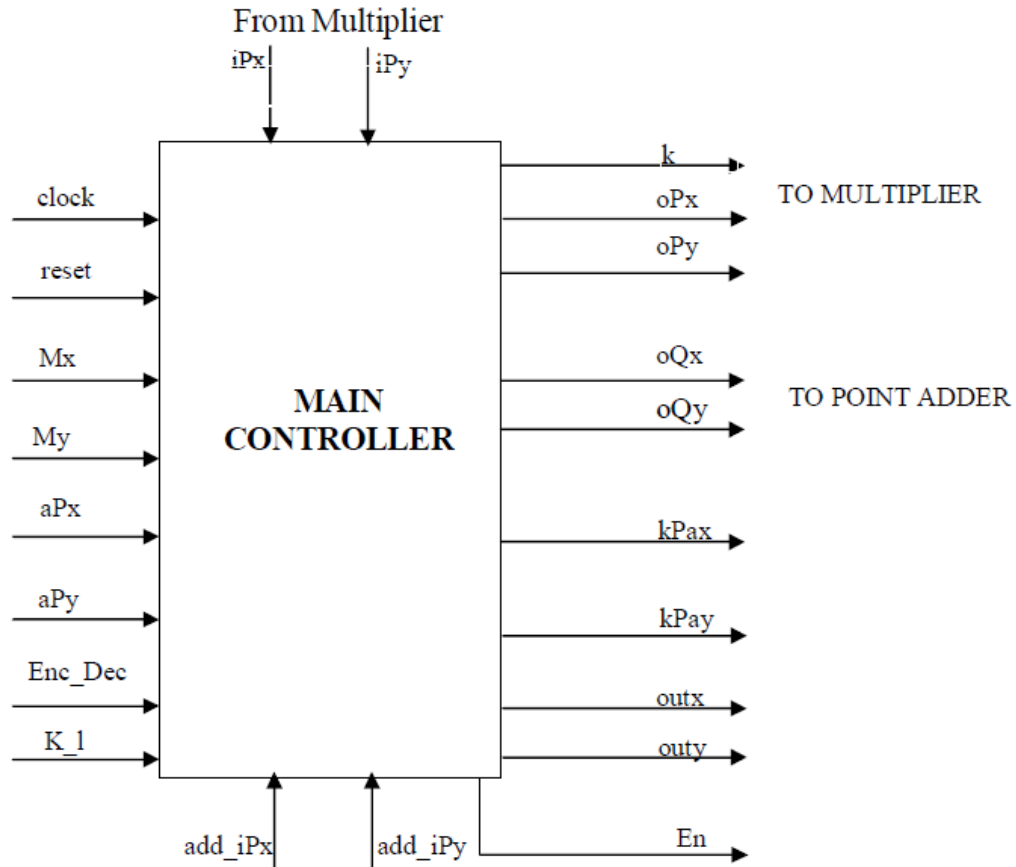


Figure 6: Block Diagram of Main Controller

The **Figure 7** shows the block diagram of multiplier. The multiplier block is selected when the *En* line is '00'. The multiplier performs multiplication of an integer with a point on the elliptic curve. The multiplication is done by successive addition. This addition follows the rules of Elliptic Curve Arithmetic, known as *Point Doubling*.

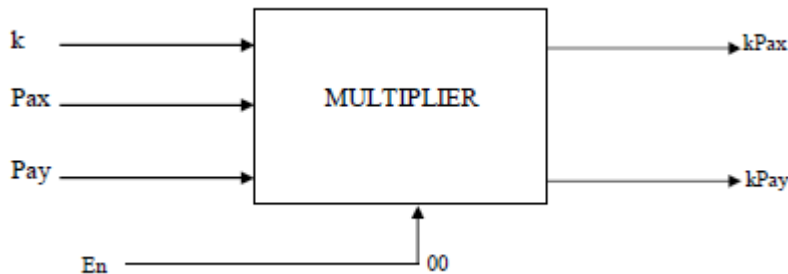


Figure 7: Block Diagram of Multiplier

From Main Controller: *k*: it is a randomly generated integer which is multiplied with a point on the curve; *Pax*: x-coordinate of a point on the elliptic curve; *Pay*: y-coordinate of a point on the elliptic curve; *En*: enables the multiplier block;

To Main Controller: *kPax*: x-coordinate of the resultant; *kPay*: y-coordinate of the resultant.

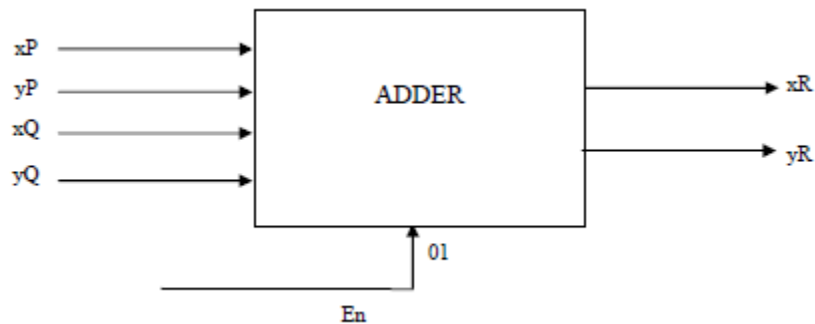


Figure 8: Block Diagram of Adder

Figure 8 shows the block diagram of adder module. The adder block is selected when the En line is '01'. The adder performs addition of two points on an elliptic curve. Addition is based on the rules of Elliptic Curve Arithmetic known as point addition. Similar to multiplication, addition is performed on projective co-ordinates and the formulae are given in the above table. The result is in affine.

From Main Controller : xP : x-coordinate of a point on the elliptic curve; yP : y-coordinate of a point on the elliptic curve; xQ : x-coordinate of a point on the elliptic curve; yQ : y-coordinate of a point on the elliptic curve; En : enables the adder block

To Main Controller : xR : x-coordinate of the resultant; yR : y-coordinate of the resultant

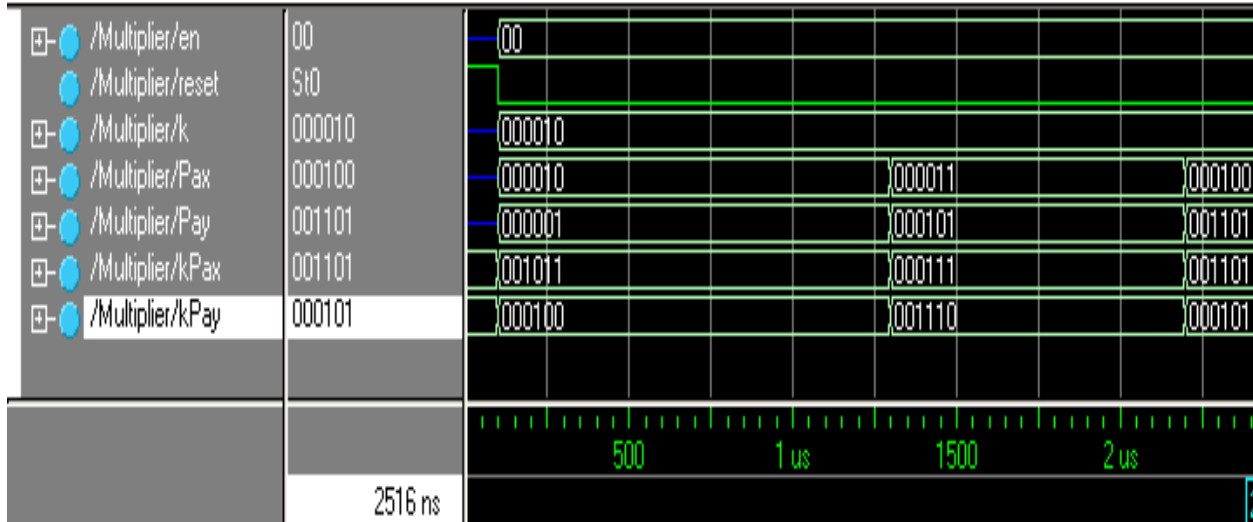
V. RESULT & DISCUSSION

Simulation Of Encryption And Decryption

In order to implement ECC cryptographic system three modules namely (i) main controller (ii) point adder and (iii) multiplier are to be simulated using Xilinx ISE simulator. The program for the above three modules are written in Verilog HDL. There are three ways to simulate a design, software simulation, hardware acceleration and hardware emulation. Software simulation is used here, which is used to run Verilog HDL based design. They load the Verilog HDL code and simulate the behaviour in software.

When the encryption_ decryption is low (0), then the encryption part is selected. The simulation waveforms of multiplier block, adder block and main controller are obtained. The simulation waveform for multiplier block is given below. When the enable line is '00' the multiplier block is selected. Then the clock is set as high and the reset as low, the system will multiply the constant K with the combination of public and private key and gives the product as KPx and KPy . The resulting output for encryption and decryption of multiplier are shown in the screen shots below in

Encryption/Multiplier:



Decryption/Multiplier :

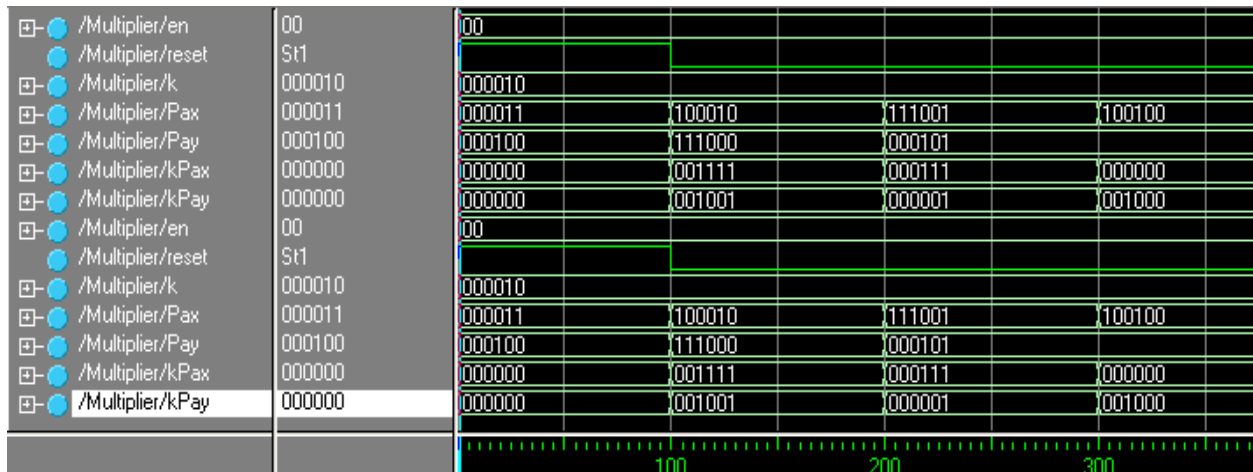
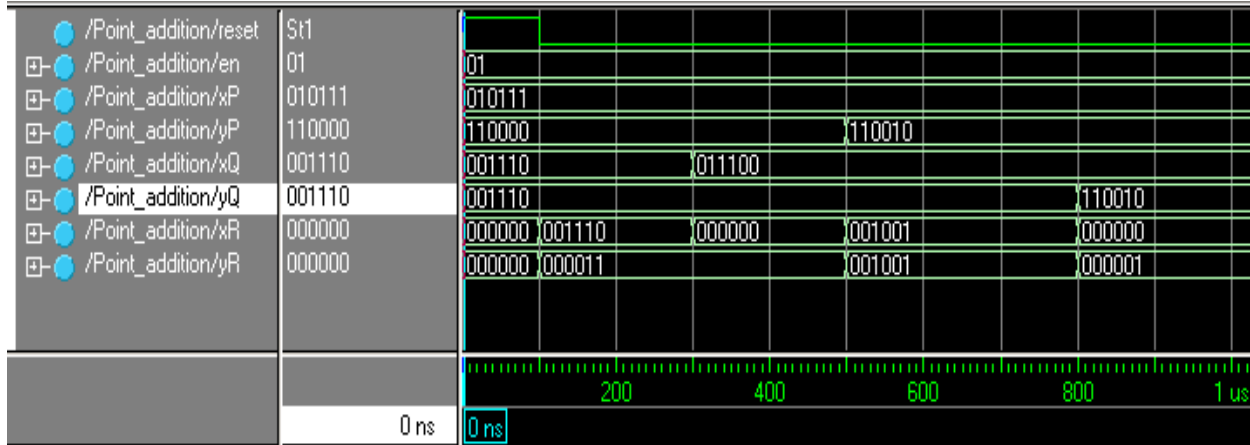


Fig.9:Screen Shots for Multiplication Operations used inEncryption and Decryption

When the enable line is '01' the adder block is selected. Then the clock is set as high and the reset as low. The adder performs the addition of two points on the elliptic curve. The message coordinates are added with the public key coordinates and the result is given to the main controller. The inputs are M_x , M_y , aP_x and aP_y and the result is M_x+aP_x and M_y+aP_y . The output screen shots obtained for the above operations are in **Figure 10**.



Decryption/Adder:

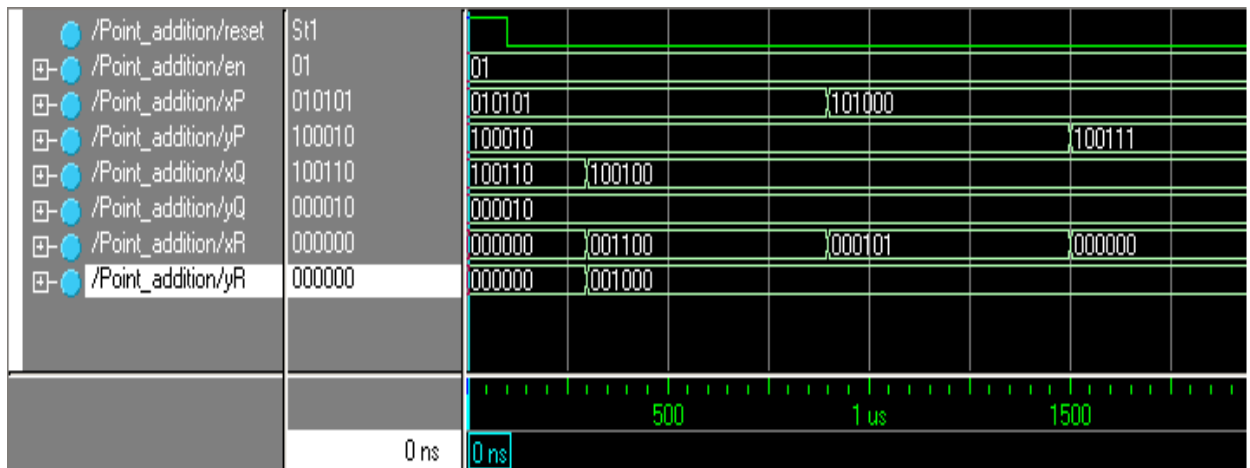


Fig.10: Output Screen Shots for Encryption and Decryption Addition Operations

The main controller controls the function of adder and multiplier. It has several internal signals for controlling the operation of the whole system. The clock is set as high and the reset is low. It has the input as the message coordinates and the public and private keys. The encrypted message coordinate is obtained as an output in the encryption process and the original message is obtained back in the decryption process. The screen shots corresponding to the main controller's output are shown in **Figure 11&12**.

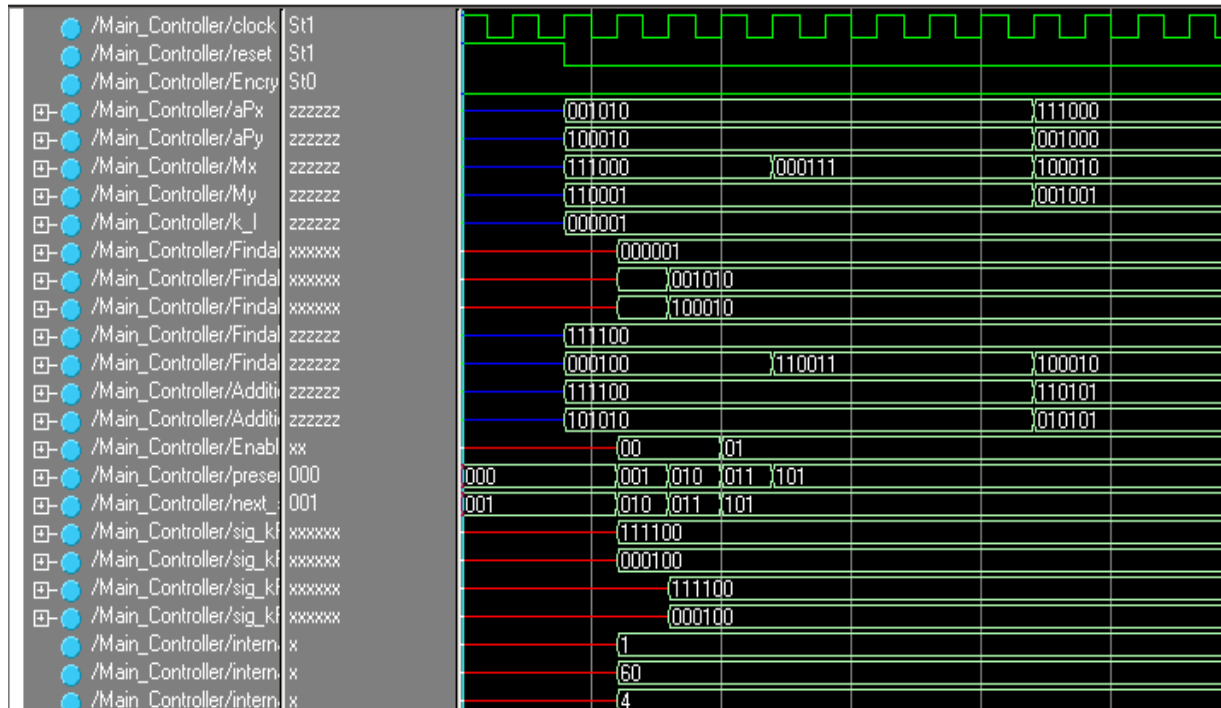


Fig.11: Main controller outputs for Encryption

Decryption/Main controller:

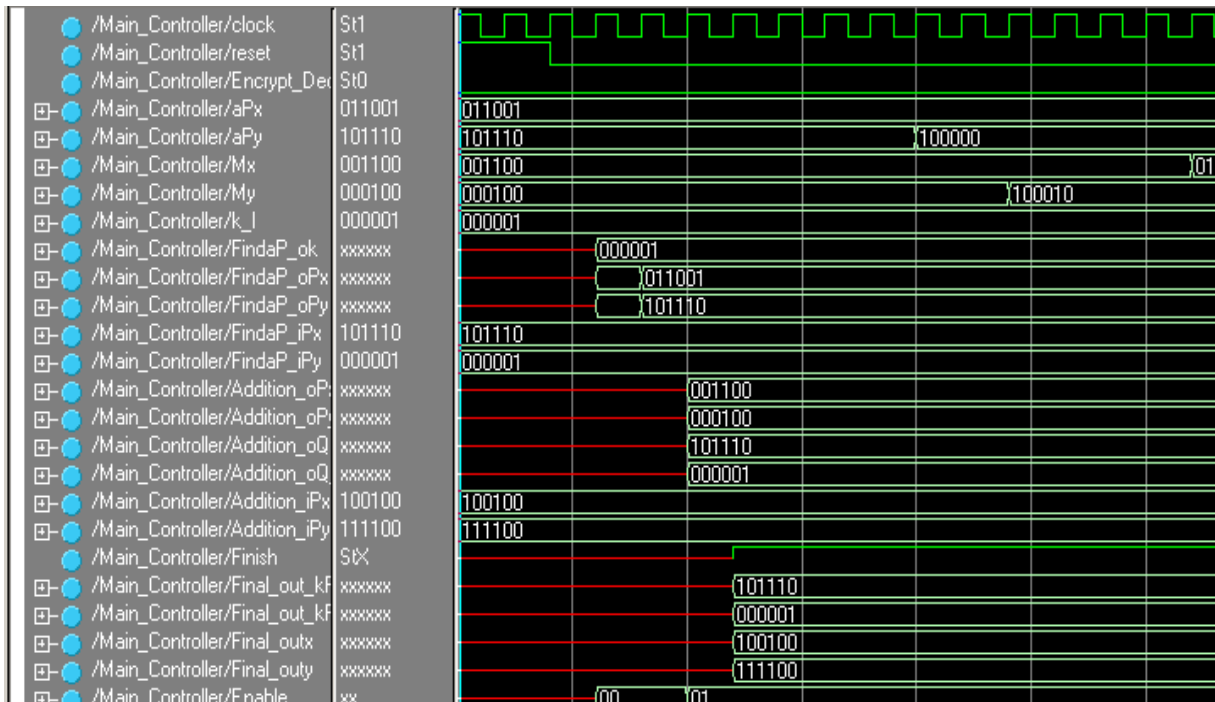


Fig.12: Main controller outputs for Decryption

Synthesis Result

The architecture has been synthesized by using Xilinx ISE. The Place and Route report from ISE shows the number of logic gates consumed. Table 1 shows the device utilization summary of encoder and decoder and Table 2 gives the timing summary

Table 1 Device Utilization Summary

SL No.	Name	Utilization
1	Multiplexer	6
2	Adder	9
3	Subtractor	9
4	Multiplier	58
5	Comparator	6
6	Latches	22
7	LUT's	43
8	IOB's	124
9	GCLK's	2
10	Flip flops	118
11	Registers	6

Table 2 Timing Summary

SL No.	Name	Utilization
1	Minimum period	5.043 ns
2	Maximum frequency	198.295 MHz
3	Total memory usage	269824 Kilobytes

VI. CONCLUSION

The Elliptic Curve Cryptography implemented in Verilog HDL provides a highly efficient encryption algorithm compared to existing algorithms such as RSA and DSA. Further It is achieved an equivalent strength with a key of lesser number of bits (150 – 200 bits). However, selecting a suitable curve and finite field are crucial for ECC security. In this work a method for implementing elliptic curve cryptography using Verilog HDL has been provided. Since the proposed hardware implementation of ECC algorithm has shorter key length, shorter parameters and shorter calculation time it proves to be efficient than other cryptographic algorithms. The simulation results provide further proof of the efficiency of the algorithm. The synthesis report gives the details upto the gate level for the logic blocks along with timing summary. The Place and route mapping for the system is achieved. Hardware architectures are much more dependent on the varying bit size than a software implementation.

VII. ACKNOWLEDGEMENTS

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REFERENCES

1. M. B. I. reaz, J. jalil, H. husian, F. H. hashim, *FPGA Implementation of Elliptic Curve Cryptography Engine for Personal Communication Systems*.
2. Shishay Welay Gebregiyorgis, *Algorithms for the Elliptic Curve Discrete Logarithm and the Approximate Common Divisor Problem*.
3. Christophe Petit, Michiel Kusters, and Ange Messeng, *Algebraic approaches for the Elliptic Curve Discrete Logarithm Problem over prime fields*.
4. W. Diffie and M. E. Hellman, "New Directions in Cryptography," *IEEE Transactions on Information Theory*, vol. 22, pp. 644–654, 1976.
5. Lyndon V. Judge, *Design Methods for Cryptanalysis*.
6. R. Rivest, A. Shamir, and L. M. Adleman, "A Method for Obtaining Digital Signatures and Public Key Cryptosystems," *Communications of the ACM*, vol. 21, pp. 120–126, 1978.
7. A. J. Menezes, P. C. van Oorschot, and S. A. Vanstone, *Handbook of Applied Cryptography. The CRC Press Series on Discrete Mathematics and its Applications*, CRC Press, first ed., 1996.
8. T. Elgamal, "A Public Key Cryptosystem and a Signature Scheme Based on Discrete Logarithms," *IEEE Transactions on Information Theory*, vol. 31, pp. 469–472, July 1985.
9. N. Koblitz, "Elliptic Curve Cryptosystems," *Mathematics of Computation*, vol. 48, pp. 203–209, January 1987.
10. D. Johnson, A. Menezes, and S. Vanstone, "The Elliptic Curve Digital Signature Algorithm (ECDSA)," *IJIS*, 2001.
11. W. Stallings, *Cryptography and Network Security*. Prentice Hall, 1999.
12. J. O'Connor and E. Robertson, "The MacTutor History of Mathematics Archive." <http://www-gap.dcs.st-and.ac.uk/history/Mathematicians/Khayyam.html>, July 1999.
13. V. S. Miller, "Use of Elliptic Curves in Cryptography," in *LNCS 218, Advances in Cryptology - CRYPTO '85* (H. Williams, ed.), (Berlin Heidelberg), pp. 417–426, *Exploratory Computer Science, IBM Research, P.O. Box 218, Yorktown Heights, NY 10598, Springer-Verlag*, 1986.
14. J. B. Fraleigh, *A First Course in Abstract Algebra*. Pearson Education (Singapore) Pte. Ltd., 2003.
15. N. Koblitz, *Algebraic Aspects of Cryptography*. Springer, 1998.
16. B. Schneier. *Applied Cryptography*. John Wiley and Sons, second edition, 1996.
17. *Cryptography and Elliptic Curves*, <http://www.tcs.hut.fi/~helger/crypto/link/public/elliptic/>
18. Julio Lopez and Ricardo Dahab, "An overview of elliptic curve cryptography", May 2000.
19. V. Miller, "Uses of elliptic curves in cryptography", *Advances in Cryptology - CRYPTO'85, LNCS 218*, pp.417-426, 1986.
20. Jeffrey L. Vagle, "A Gentle Introduction to Elliptic Curve Cryptography", *BBN Technologies*
21. Mugino Saeki, "Elliptic curve cryptosystems", *M.Sc. thesis, School of Computer Science, McGill University*, 1996. <http://citeseer.nj.nec.com/saeki97elliptic.html>
22. J. Borst, "Public key cryptosystems using elliptic curves", *Master's thesis, Eindhoven University of Technology*, Feb. 1997. <http://citeseer.nj.nec.com/borst97public.html>
23. Aleksandar Jurisic and Alfred Menezes, "Elliptic Curves and Cryptography", *Dr. Dobb's Journal*, April 1997.
24. Robert Milson, "Introduction to Public Key Cryptography and Modular Arithmetic"
25. Aleksandar Jurisic and Alfred J. Menezes, *Elliptic Curves and Cryptography*
26. William Stallings, *Cryptography and Network Security-Principles and Practice second edition*, Prentice Hall publications.
27. R. Schroppel, H. Orman, S. O'Malley and O. Spatscheck, "Fast key exchange with elliptic key systems", *Advances in Cryptography, Proc. Crypto '95, LNCS 963*, pp. 43-56, Springer-Verlag, 1995.
28. M. Rosing, *Implementing Elliptic Curve Cryptography*. Manning Publications, 1999.